Structure Functions at low Q^2

- ➔ experiment e99-118
- ➔ experiment e00-002
- ➔ experiment e94-110
- ➔ two photon effects

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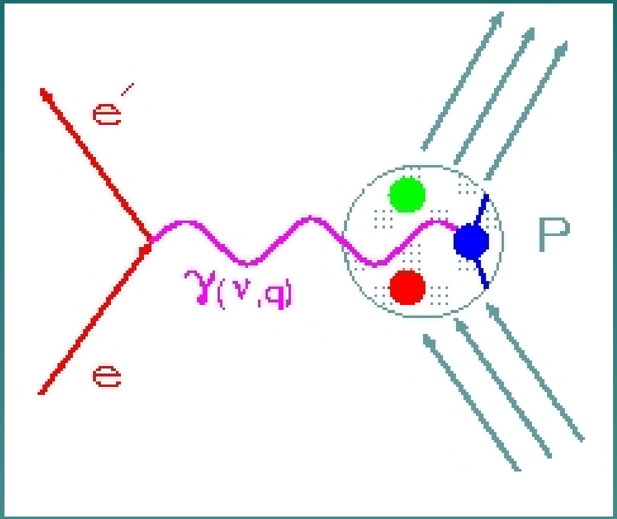
1. e99-118

Inclusive e + p → e + X Scattering



Rosenbluth:

$$\frac{d\sigma}{d\Omega dE'} = \Gamma(\sigma_T + \varepsilon\sigma_L)$$

Born Approximation

Where: Γ = flux of transversely polarized virtual photons
ε = relative longitudinal polarization

Alternatively:

$$\frac{d\sigma}{d\Omega dE'} = \sigma_{mott}\left(F_2/\nu + 2F_1\tan^2(\theta/2)/M\right)$$

Transverse

$$R = \frac{\sigma_L}{\sigma_T} = \frac{F_L}{2xF_1} \quad F_L = \left(1 + \frac{4M^2x^2}{Q^2}\right)F_2 - 2xF_1$$

longitudinal

Mixture

Physics at low Q^2 (Overview)

$$\boxed{W^{\mu\nu} = \frac{F_1(x,Q^2)}{M}\left(-g_{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + \frac{F_2(x,Q^2)}{M(p\cdot q)}\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right)}$$

$$W^{\mu\nu} = -\frac{F_1(x,Q^2)}{M}g^{\mu\nu} + \frac{F_2(x,Q^2)}{M(p\cdot q)}p^\mu p^\nu$$

$$+\underbrace{\left(\frac{F_1(x,Q^2)}{M} + \frac{F_2(x,Q^2)}{M}\frac{p\cdot q}{q^2}\right)}\frac{q^\mu q^\nu}{q^2}$$

$$\underbrace{-\frac{F_2}{M}\frac{p^\mu q^\nu + p^\nu q^\mu}{q^2}}$$

at $Q^2 \rightarrow 0$:

$$F_2 = O(Q^2)$$

$$\frac{F_1}{M} + \frac{F_2}{M}\frac{p\cdot q}{q^2} = O(Q^2)$$

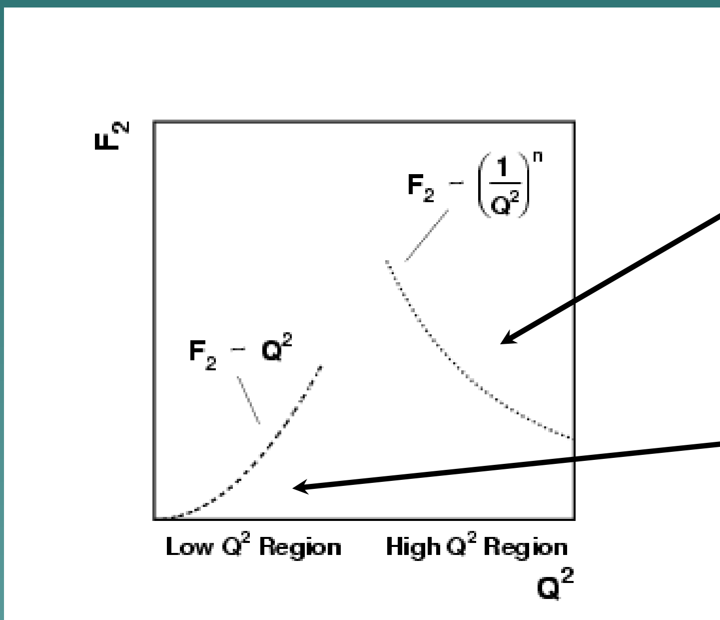
$$R(x,Q^2) = \frac{\sigma_L}{\sigma_T} = \frac{(1+4M^2x^2/Q^2)F_2}{2xF_1} - 1 \xrightarrow{Q^2 \rightarrow 0} 0$$

Physics at low Q^2 (Overview)

- $F_2(x,Q^2) \to Q^2$ as $Q^2 \to 0$
- $F_L(x,Q^2) \to Q^4$ as $Q^2 \to 0$
- $R(x,Q^2) = \sigma_L/\sigma_T \to Q^2$ as $Q^2 \to 0$
- $\sigma^{\gamma p} = \sigma_T + \varepsilon\sigma_L \to \sigma_T$ as $Q^2 \to 0$

Twist Effects

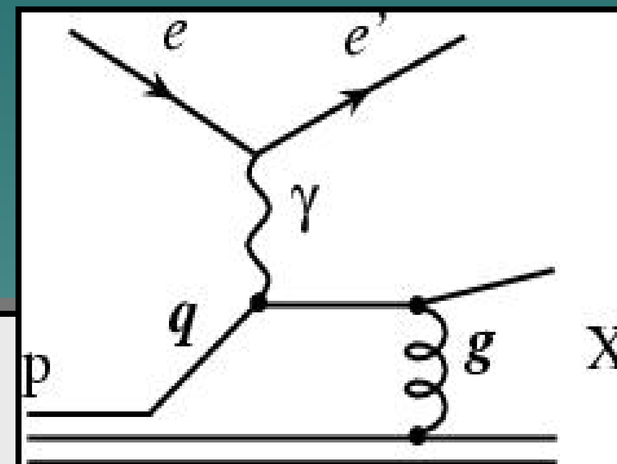
$$F_2(x,Q^2)=\sum_{n=0}^{\infty}\frac{C_n(x,Q^2)}{\left(Q^2\right)^n}=C_0(x,Q^2)+\underbrace{\frac{C_1(x,Q^2)}{Q^2}+\frac{C_2(x,Q^2)}{Q^4}+\ldots\ldots\ldots}$$



at $Q^2 \rightarrow 0$:

$$F_2 = O(Q^2)$$

$$\frac{F_1}{M}+\frac{F_2}{M}\ \frac{p\cdot q}{q^2}=O(Q^2)$$

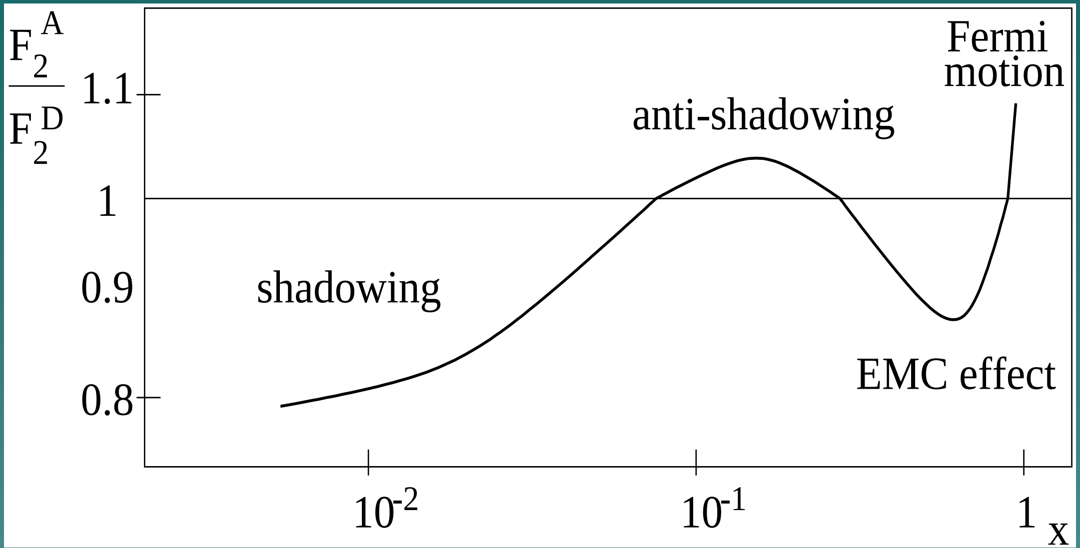


F$_2$ is usually parametrized as:

$$F\left(x,Q^2\right)=F^{LT}\left(x,Q^2\right)\left(1+\frac{C(x)}{Q^2}+\ldots\right)$$

C(x) characterizes the strength of the twist-four term

Nuclear Effects in F_2 and R



x < 0.05-0.1 Shadowing
x ≈ 0.1-0.2 Anti-Shadowing
0.2-0.3 < x < 0.8 EMC Effect
x > 0.8 Fermi Motion

$$\frac{\sigma_A}{\sigma_D} = \frac{F_2^{A}(1+\varepsilon R_A)(1+R_D)}{F_2^{D}(1+R_A)(1+\varepsilon R_D)}$$

$$\underbrace{\frac{\sigma_A}{\sigma_D} \underset{\mathrm{IF}}{=} \frac{F_2^{A}}{F_2^{D}}}$$

ε=1 or $R^A=R^D$

$$R = \frac{\sigma_L}{\sigma_T}$$

ε ≈ 1 (for high Energy) virtual photon polarization parameter

A – dependence of R at low Q^2 ?

Two Methods of Getting R

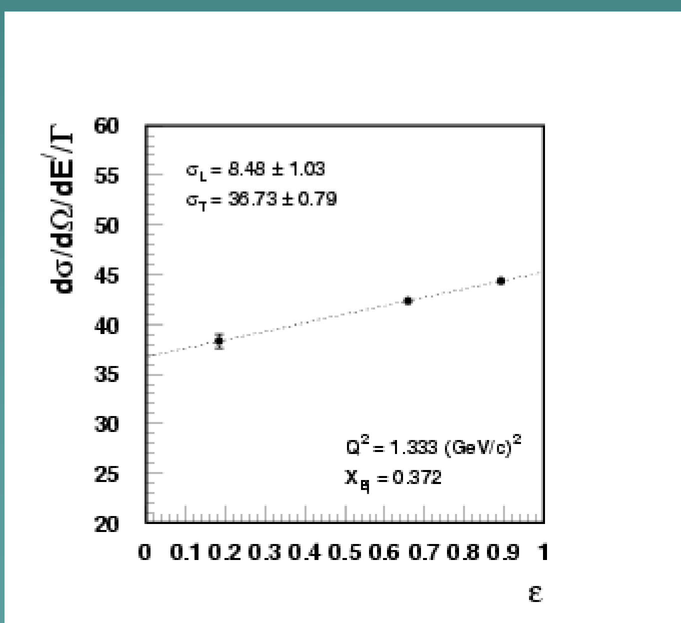
1st Method

Rosenbluth Separation

Requirements:

The same x, Q^2 but different ε.

$$\frac{d^2\sigma}{d\Omega dE'} = \Gamma(\sigma_T + \varepsilon\sigma_L)$$



2nd Method

Model Dependent Method

Requirements: **Good model for $F_2(x,Q^2)$**

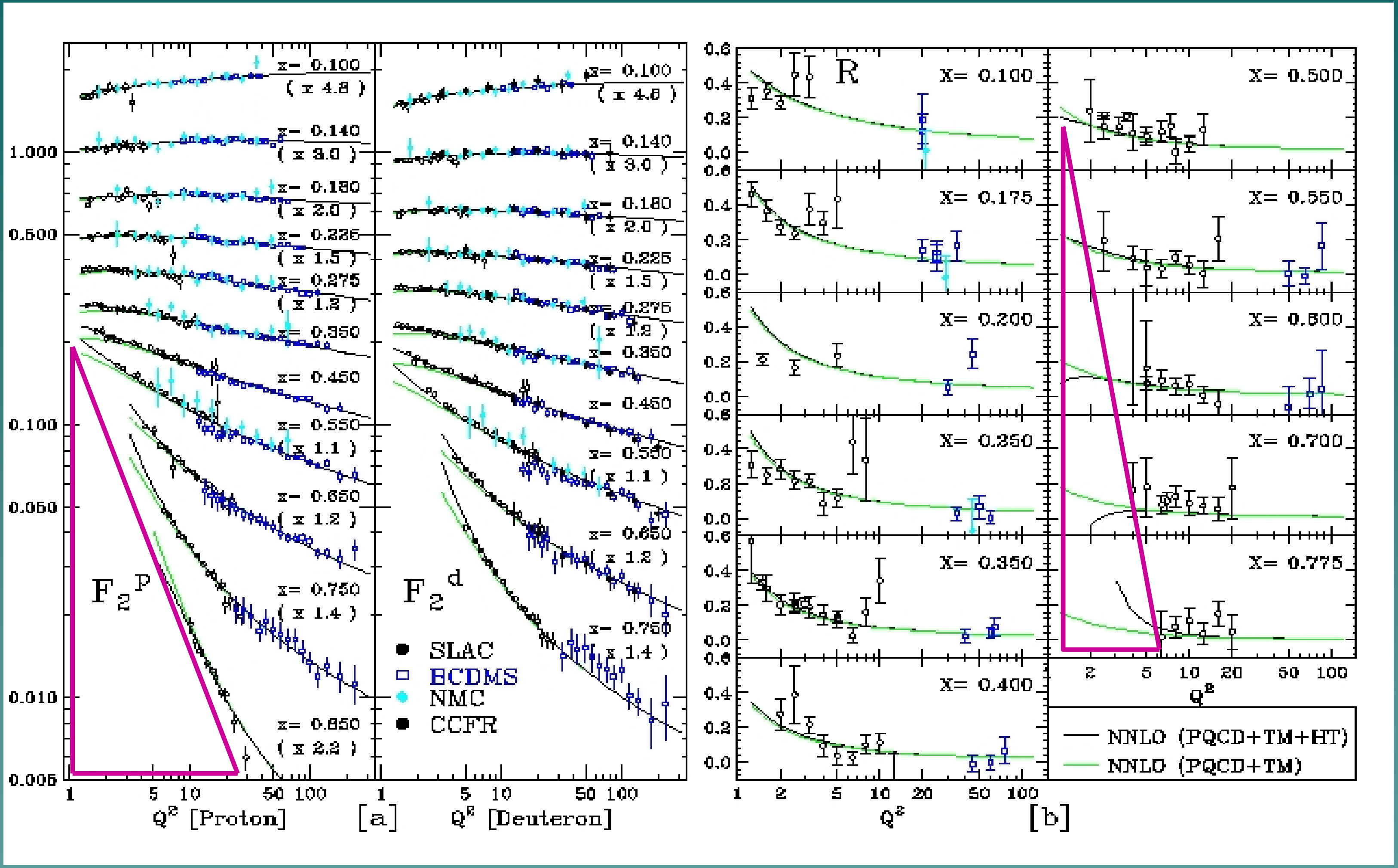
Using σ_{exp} and $F_2(x,Q^2)$ model, R can be calculated.

Model (F2ALLM)

$$\frac{d\sigma}{dE'd\Omega} = \sigma_{Mott}\frac{2m_p x F_2}{Q^2\varepsilon}\left(\frac{1+\varepsilon R}{1+R}\right)$$

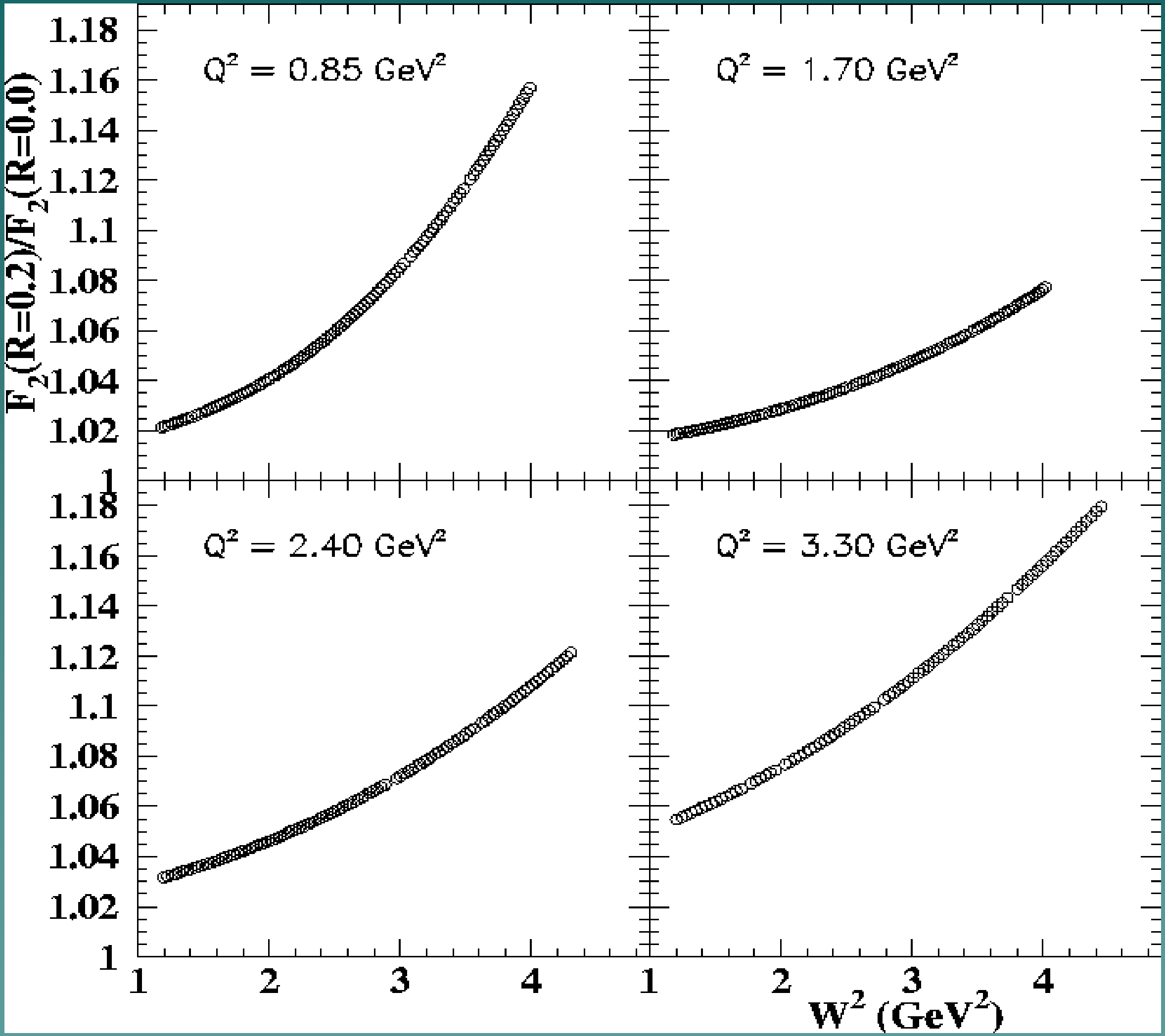
σ^{exp}

Experimental Status of Unpolarized SFs



- ➔ F_2 well measured - responsible for much understanding of proton structure
- ➔ Nonetheless, large *x*, low Q^2 region is sparse
- ➔ R (F_L), is not at all so well measured (especially large *x*, low Q^2)
- ➔ Situation is worse for nuclei
- ➔ *If R nonzero,* NEED longitudinal / transverse (L/T) separations to extract F_2

F_2 Sensitivity on R

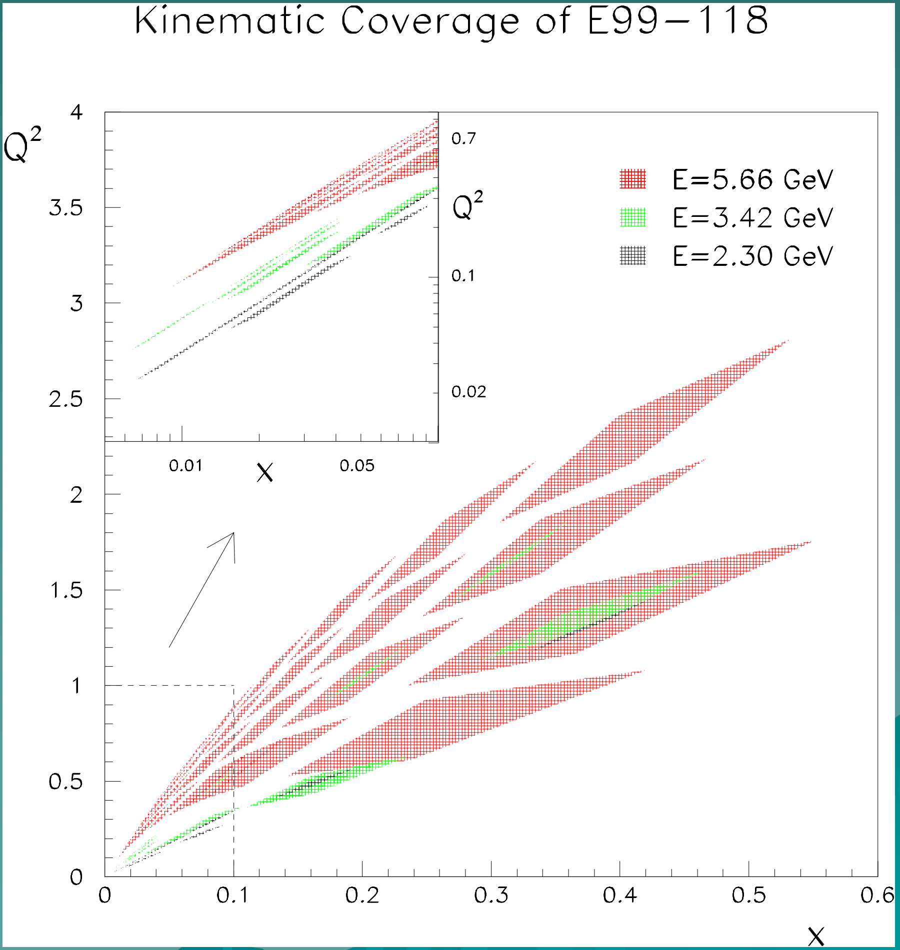


At W^2 = 4 GeV^2 and Q^2 < 1 GeV^2, F_2 will vary by 15% depending on the choice of R = 0 or R = 0.2. At higher Q^2, this can be as much as 20%.

Kinematical Coverage of e99-118

$e^- A \rightarrow e^{-\prime} X$

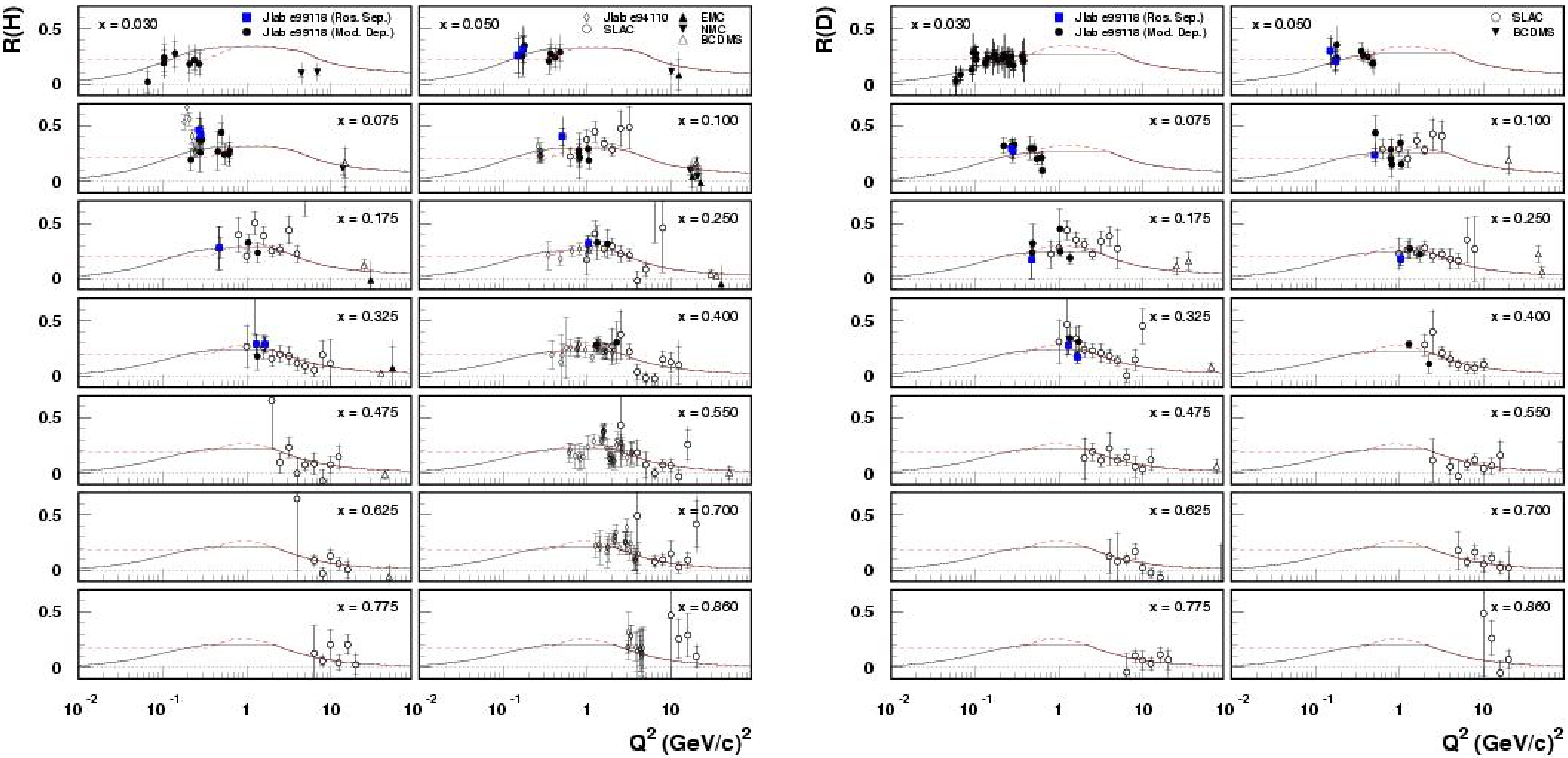
№	Beam Energy	Min E'	Max E'	Min θ°	Max θ°
1	5.648	0.418	5.132	10.60	22.60
2	3.419	0.440	3.220	10.60	52.00
3	2.301	0.440	1.950	10.60	69.00



Targets: H, D, Al, C, Cu, Au

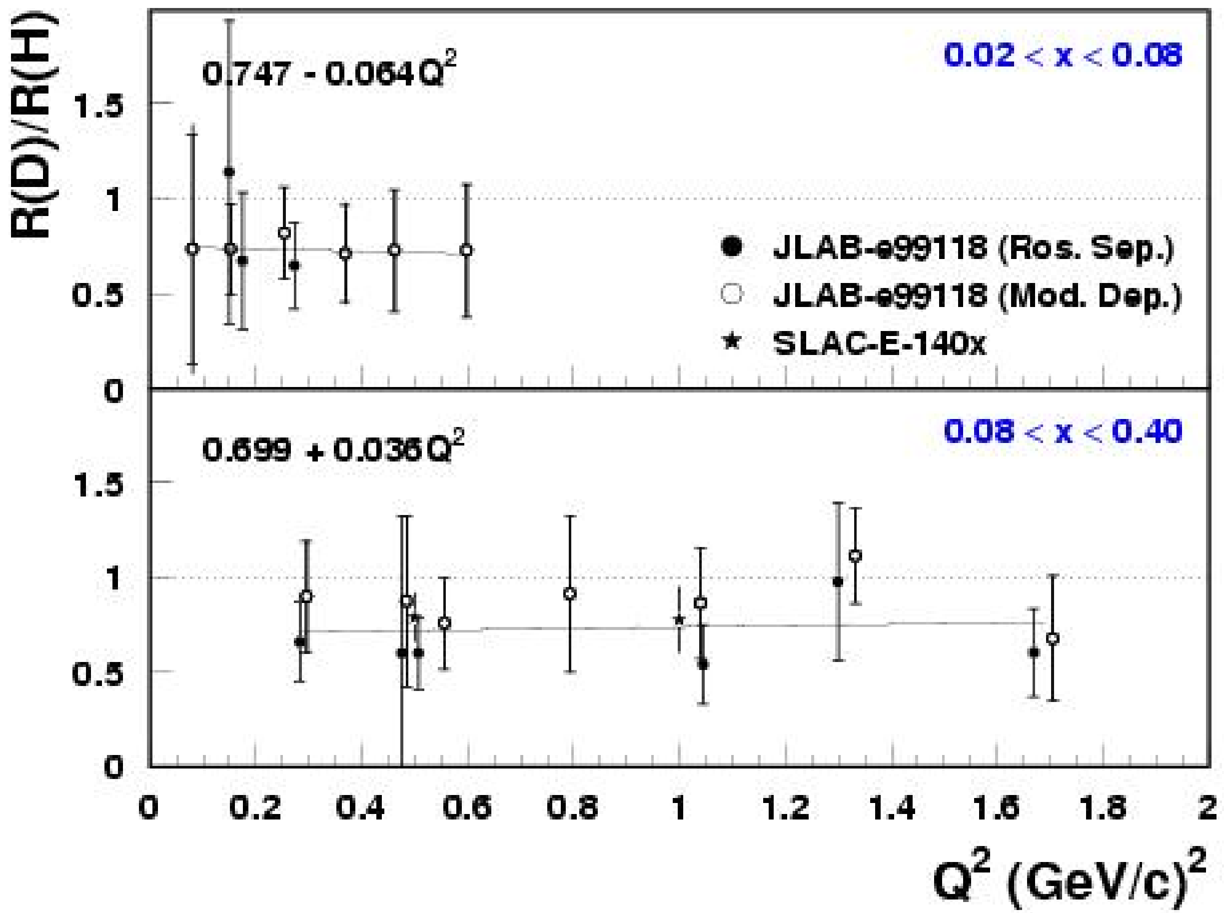
Results from e99-118

$R=\sigma_L/\sigma_T$



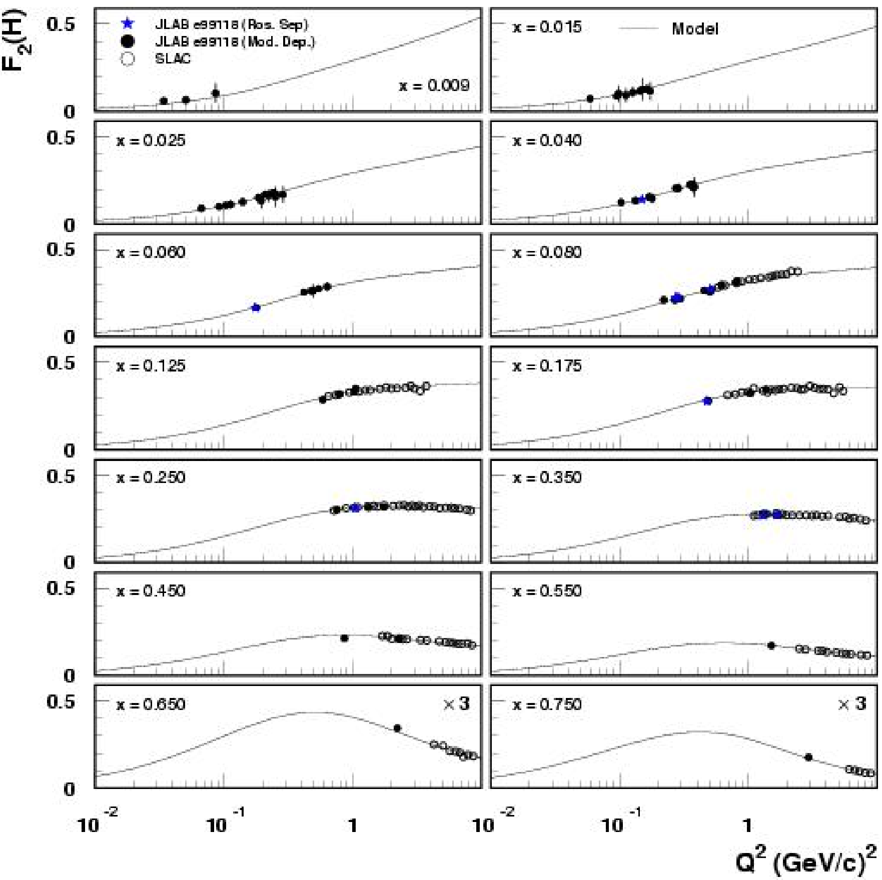
Results from e99-118

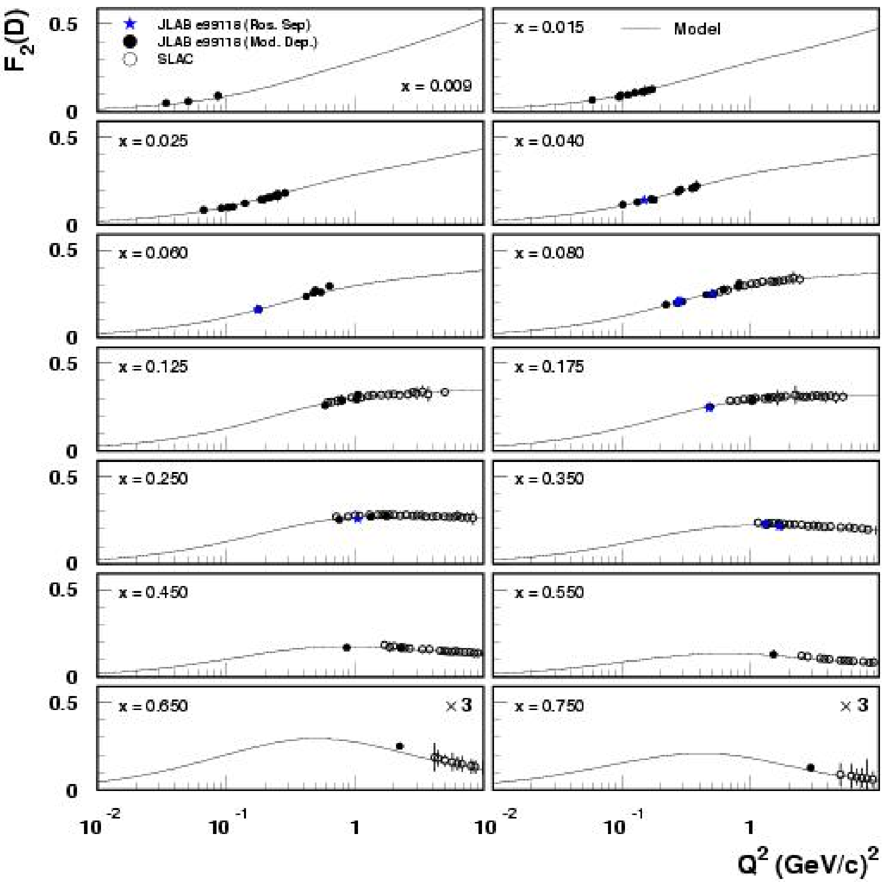
Ratio of R^D/R^H



Results from e99-118

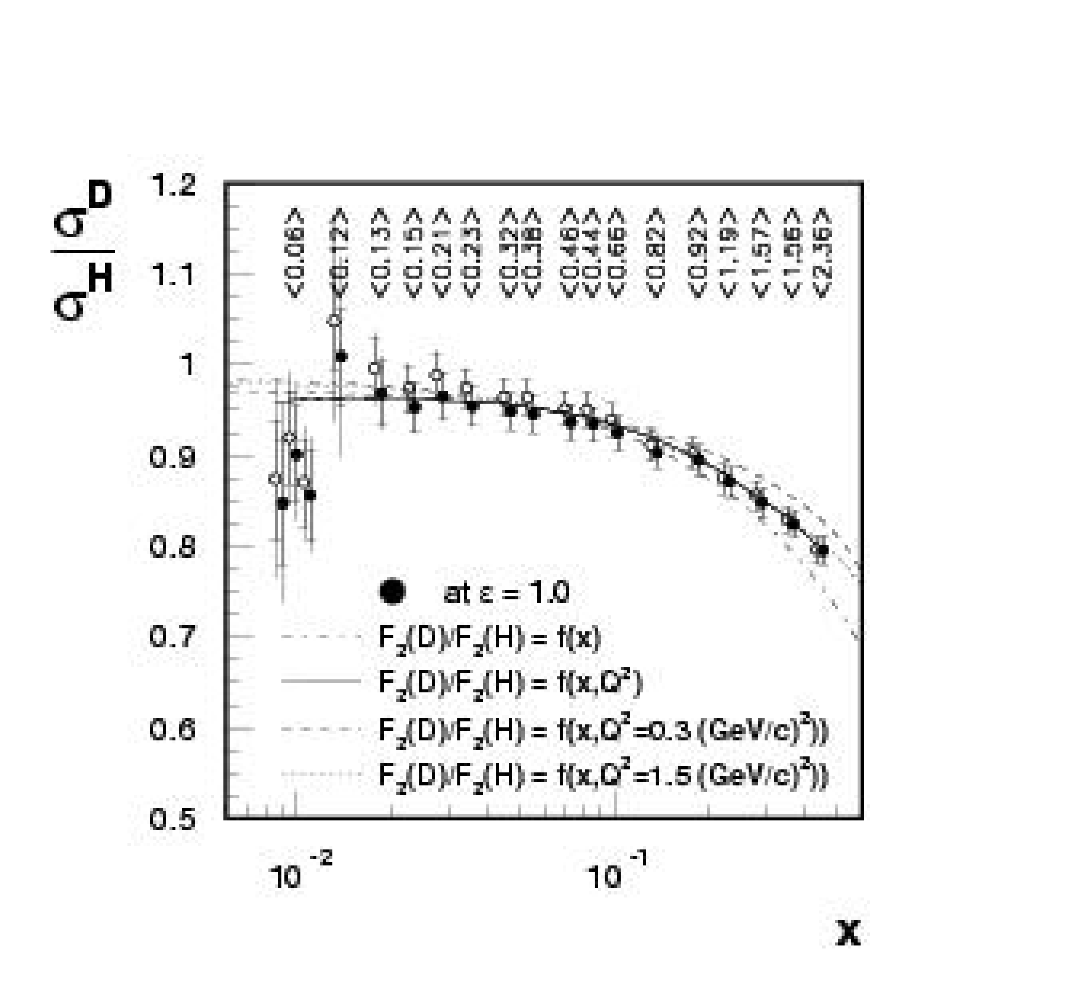
Ratio of F_2^H & F_2^D





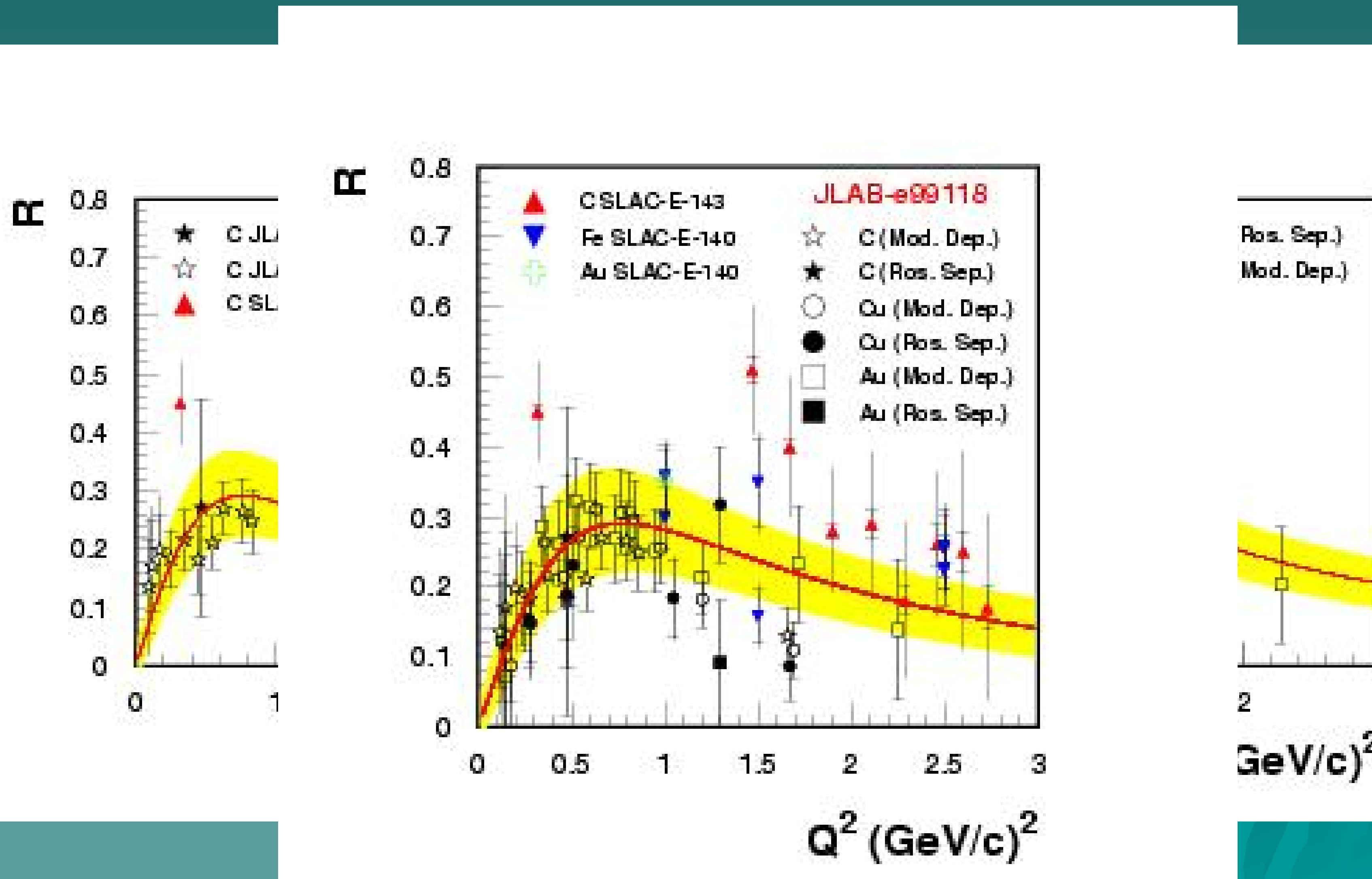
Results from e99-118

Ratio of σ^D/σ^H (F_2^D/F_2^H)



Results from e99-118

$R = \sigma_L / \sigma_T$



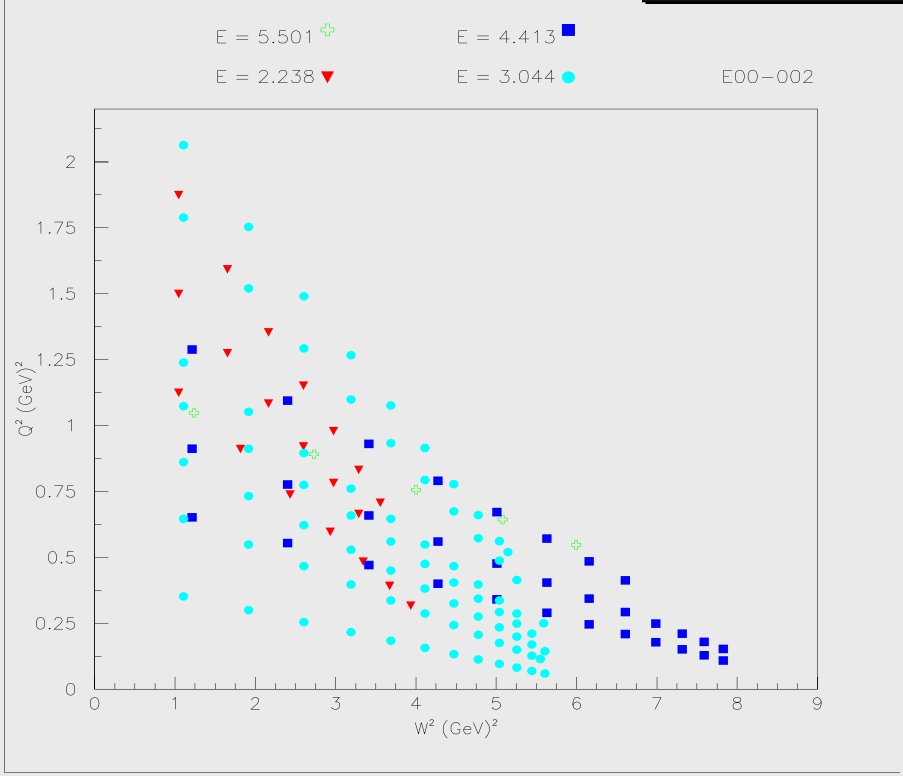
2. e00-002

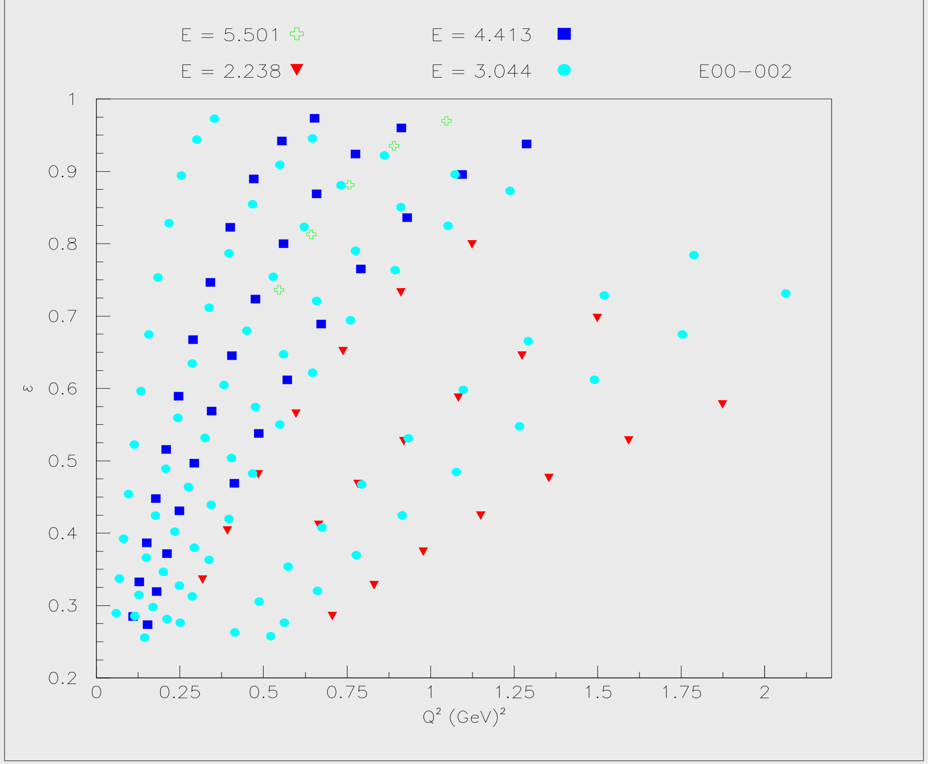
Kinematics for cross section and L/T separation

$0.05 < Q^2 < 1.7$

$0.88 < W^2 < 8.0$

$2.2 < E < 5.5$

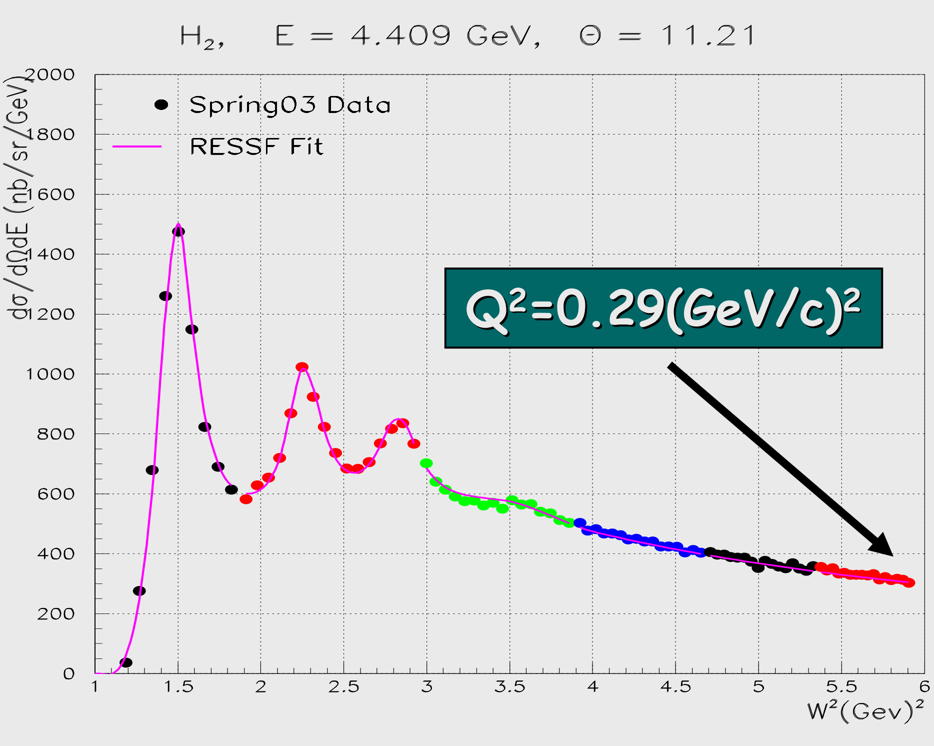


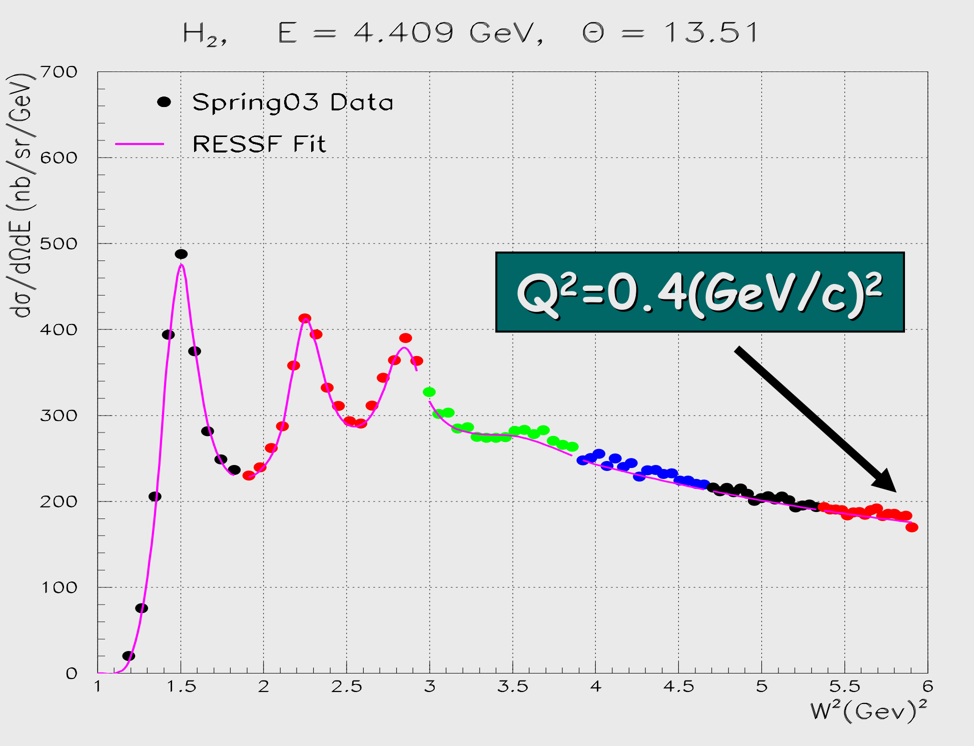


hydrogen and deuterium targets

Status of e00-002

- ▶ Calibrations (Yes)
- ▶ Efficiencies (Yes)
- ▶ Background Events (No/Yes)
- ▶ Cross Sections (No/Yes)
- ▶ Rad. Corrections (No/Yes)





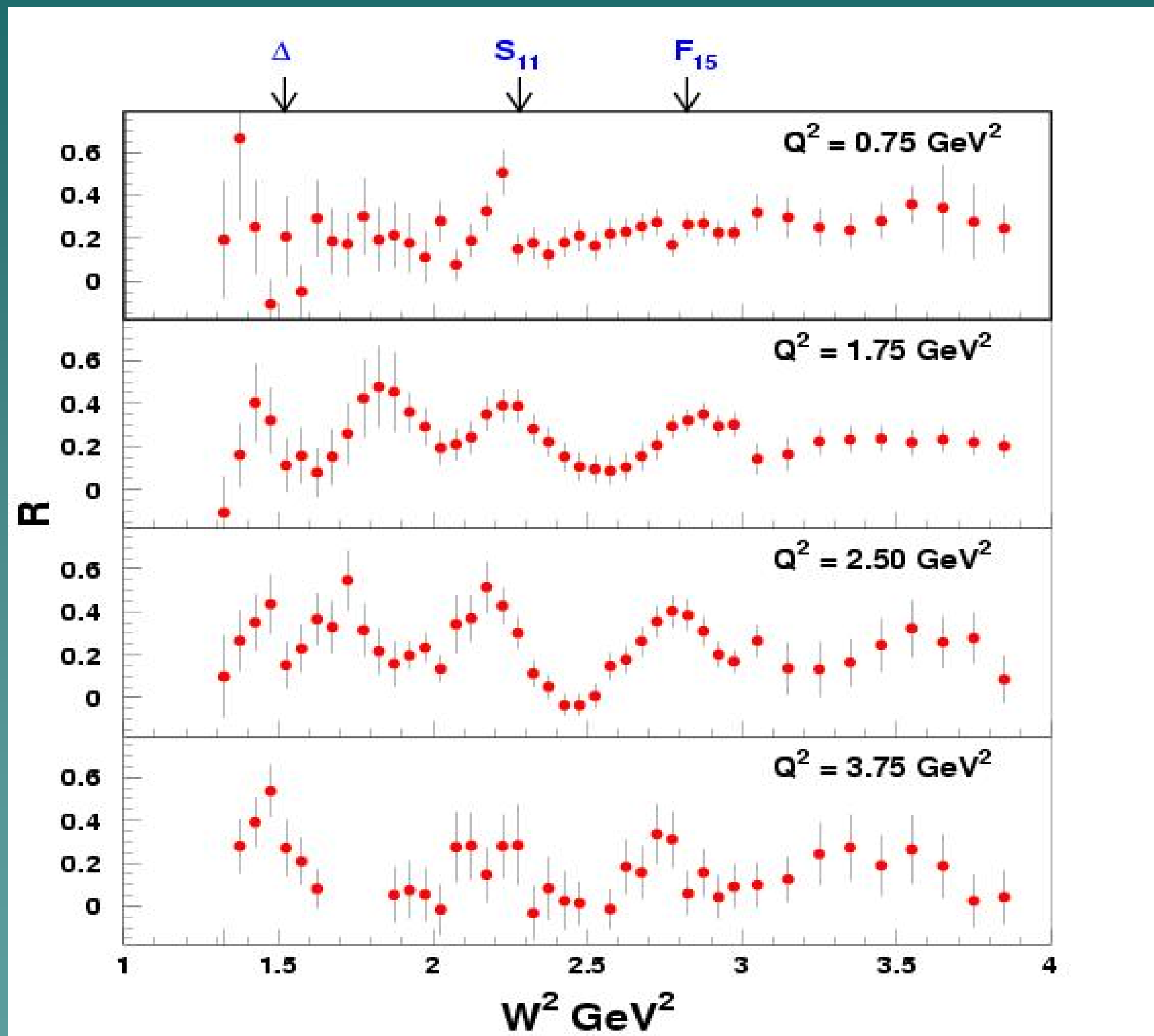
3. Two Photon-Effects

(experiment e94-110)

Results from e94-110

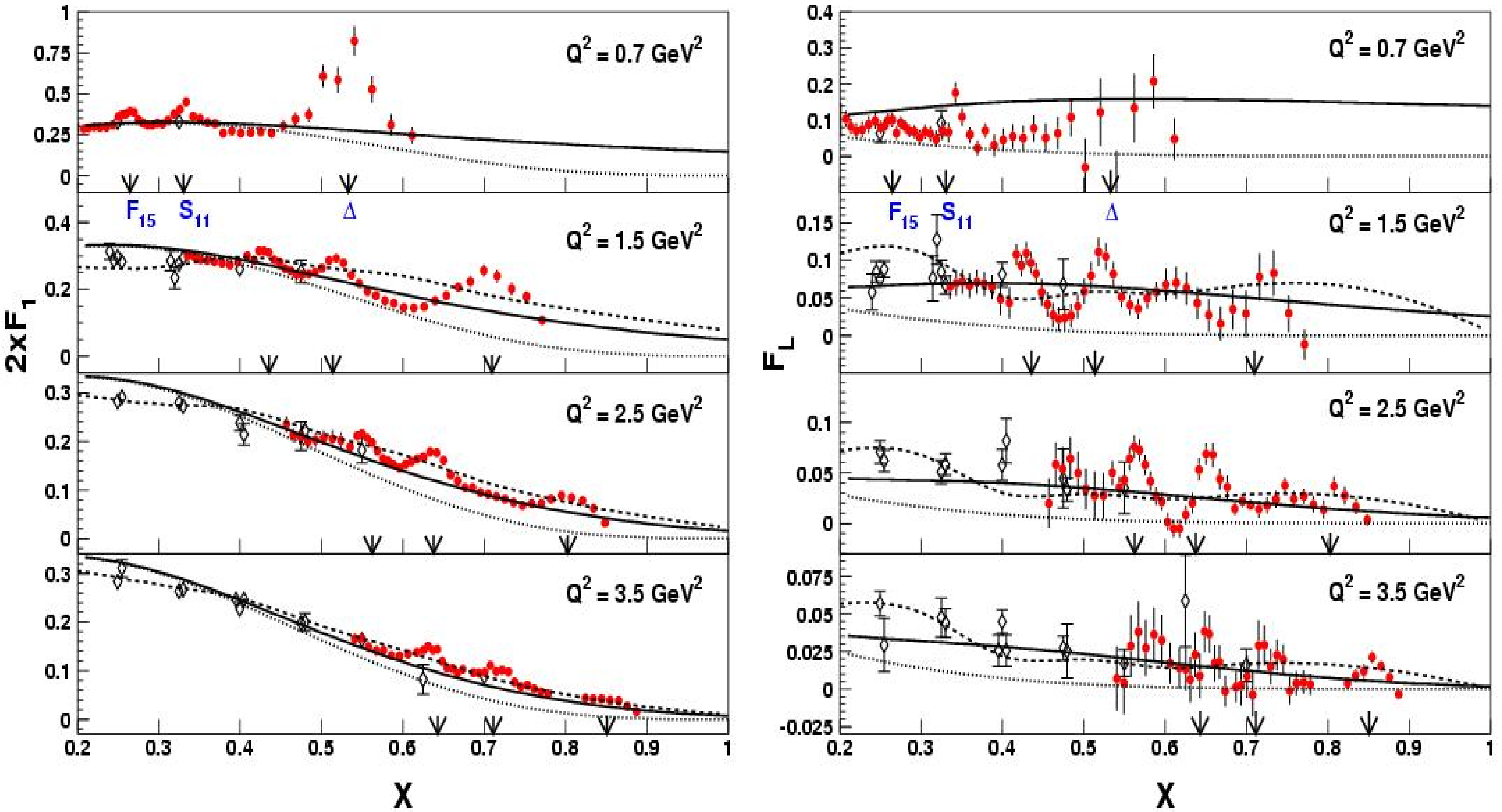
Systematic Uncertainty

$R = \sigma_L/\sigma_T$



Results from e94-110

(F_1 & F_L)



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Two Methods of Form Factors Measurements and Two Different Results

The Rosenbluth Separation Method may be used to extract the form factors G_E and $G_{M,}$ and hence ratio $(G_E/G_M)^2$ from the ε dependence of a reduced elastic cross section at fixed Q^2

$$\sigma_r \equiv \frac{d\sigma}{d\Omega}\frac{\varepsilon(1+\tau)}{\sigma_{Mott}} = \varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right) \qquad \tau = \frac{Q^2}{4M^2}$$

τG_M^2 - intercept G_E^2 - slope

With Increasing Q^2, the cross section is dominated by $G_{M,}$ while the relative contribution of the G_E term is diminished

Polarization Transfer Method

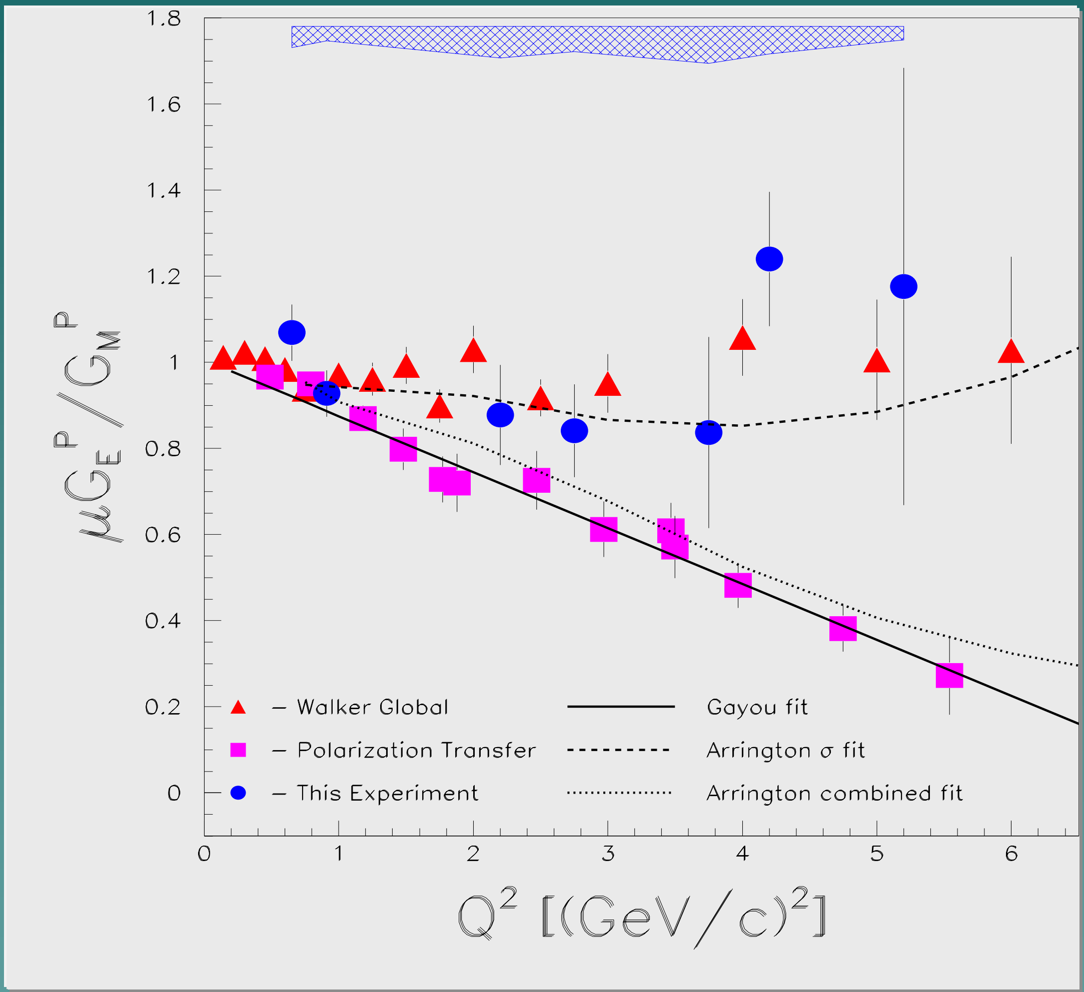
$$\frac{G_E}{G_M} = \frac{P_t}{P_l}\frac{\left(E + E^{/}\right)\tan\left(\frac{\theta_e}{2}\right)}{2M}$$

P_t – transverse component of the final proton polarization

P_l – longitudinal component of the final proton polarization

θ_e – angle between the initial and final directions of the lepton.

Two Methods of Form Factors Measurements and Two Different Results



Large Discrepancy currently exist between the ratio of electric to magnetic proton form factors extracted from previous cross section measurements (R ≈ 1), and that recently measured via polarization transfer in Hall A Jlab, (R ≈ 1-0.13).

1. R. C. Walker et al., Phys. Rev. D49, 5671 (1994).

2. M. Jones et al., Phys. Rev. Lett. 84, 1398 (2000).

3. O. Gayou et al., Phys. Rev. Lett. 88, 092301 (2002); O. Gayou et al., Phys. Rev. C64, 038202 (2001).

4. M. E. Christy et al., Phys. Rev. C70, 015206 (2004)

❖ Possible contribution from 2-photons exchange which are not fully accounted for in the standard radiative corrections procedure of Mo-Tsai could explain the discrepancy.

Experiments Included in the Analysis

Elastic Data	Q^2 $(Gev/c)^2$	№ of L-Ts	$\delta\sigma/\sigma$	Lab
Janssents et al.	0.2-0.9	20	4.7 %	Mark III
Litt et al.	2.5-3.8	4	1.7 %	SLAC
Berger et al.	0.4-1.8	8	2.6 %	Bonn
Walker et al.	1.0-3.0	4	1.1 %	SLAC
Andivahis et al.	1.8-5.0	5	1.3 %	SLAC
Chrusty et al.	0.9-5.2	7	1.3 %	Jlab
Qattan et al.	2.6-4.1	3	0.6 %	Jlab

Inelastic Data	W^2 Gev^2	№ of L-Ts	$\delta\sigma/\sigma$	Lab
Liang et al.	1.3-1.9	191	1.7 %	Jlab
Dasu et al.	3.2-30	61	3.0 %	SLAC

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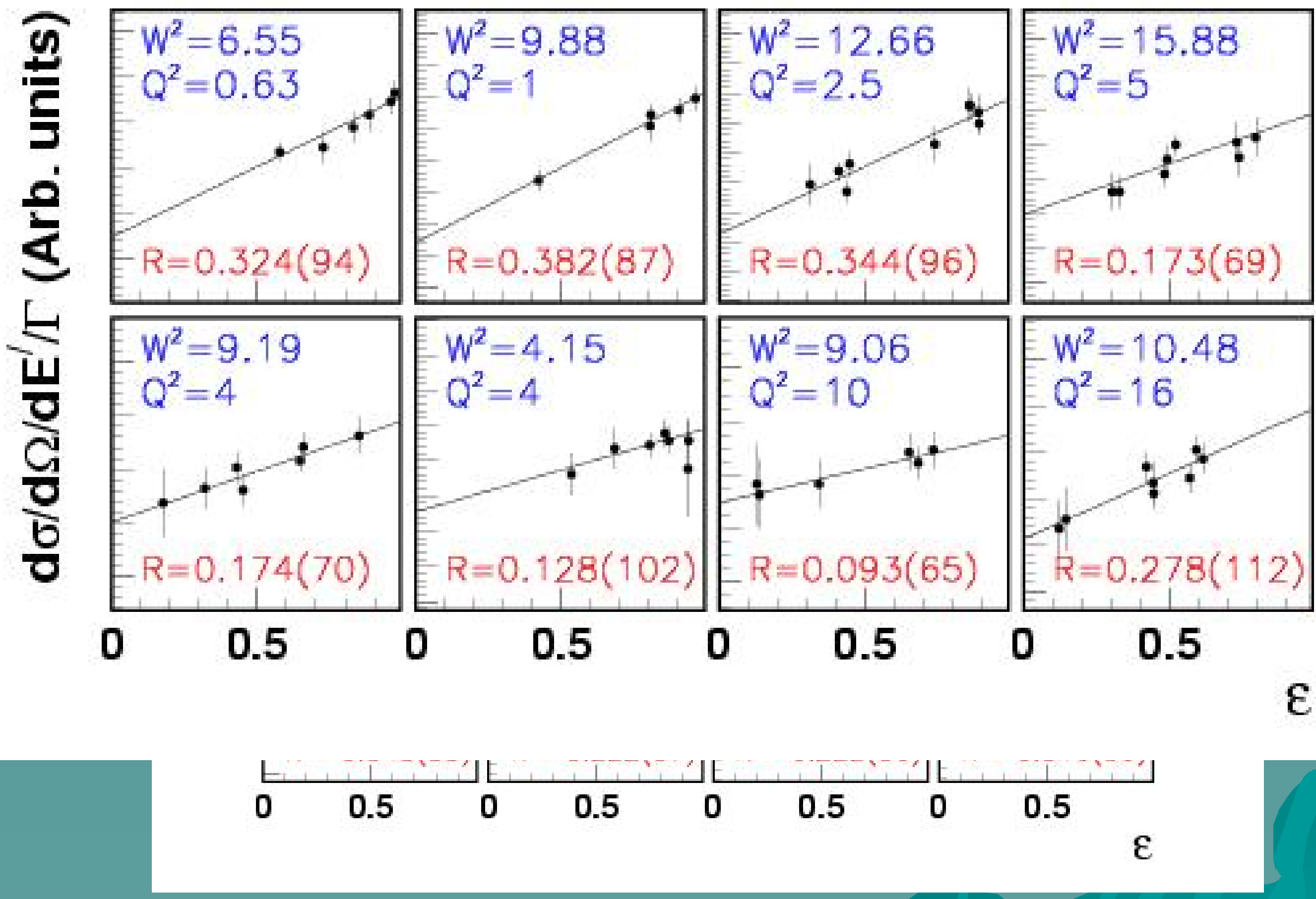
Example Rosenbluth Separations

Experiment e94-110, 191 LT Separations

Experiment e140, 61 LT Separations



Analysis and Results (part-1)

Each data set at fixed Q^2 and W^2 has been fitted with form:

$$\sigma_r = P_0 \cdot \left[1 + P_1(\varepsilon - 0.5) + P_2(\varepsilon - 0.5)^2\right]$$

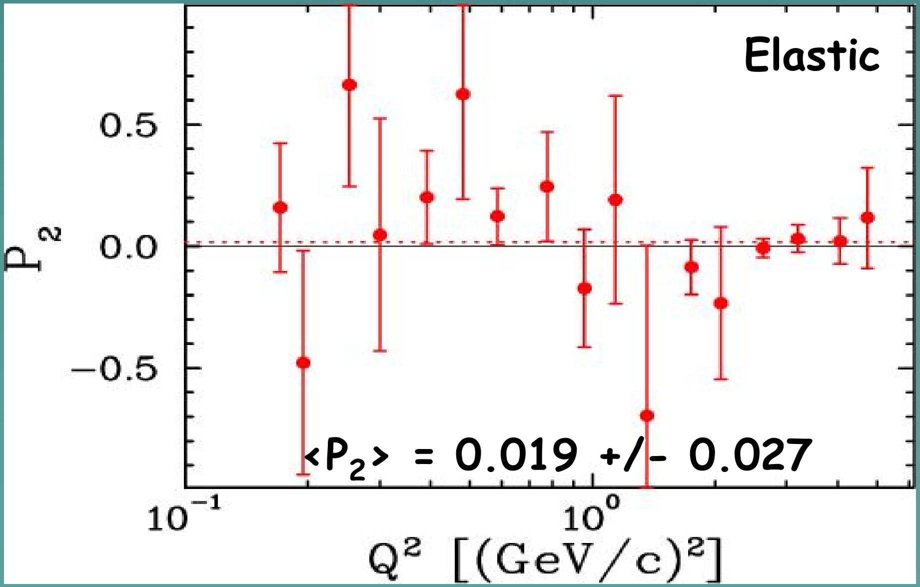
In the absence of TPE, $P_0=\sigma_T+0.5\sigma$, $P_1=\sigma_L$, and $P_2=0$

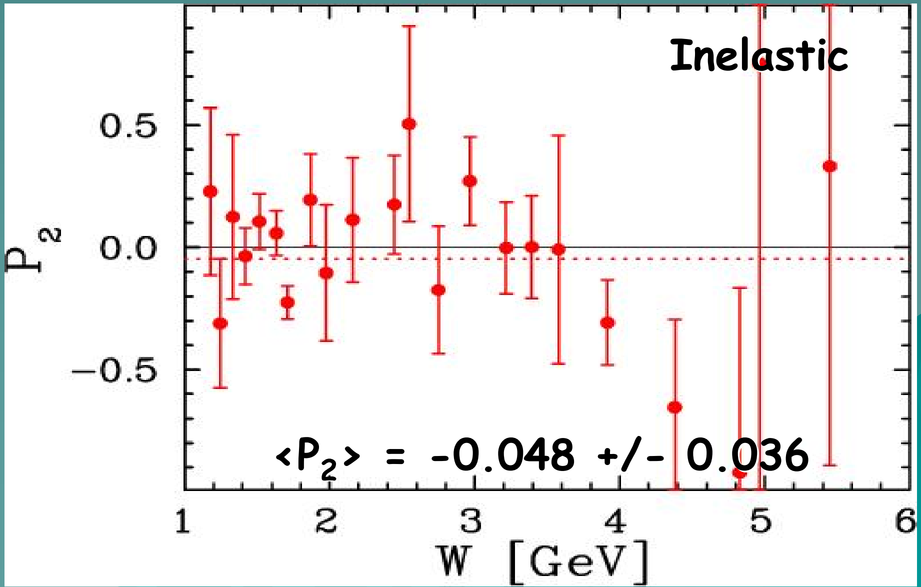
TPE corrections can modify P_0 and P_1, and may introduce a non-zero value of P_2, the fractional curvature relative to the P_0, the cross section at ε = 0.5.

While P_2 represents the fractional curvature, the size of cross section deviations from linearity will be much smaller. For P_2=10%, the maximum deviation of the cross section from P_2=0 would be 2.5% at ε=0,1. The effect are even smaller if the ε range of the data, $\Delta\varepsilon$, is less than one.

Maximum observed deviation from Linearity

$$\Delta_{max} = \frac{(\sigma - \sigma_{fit})_{max}}{\sigma} \approx \frac{P_2(\Delta\varepsilon)^2}{8}$$





Analysis and Results (part-2)

Region	$\langle P_2 \rangle$	$\lvert P_2\rvert_{MAX}$ 95% CL	Δ_{max} 95% CL
Elastic	0.019(27)	0.064	$0.8\%(\Delta\varepsilon)^2$
Resonance	-0.060(42)	0.086	$1.1\%(\Delta\varepsilon)^2$
DIS	-0.012(71)	0.146	$1.8\%(\Delta\varepsilon)^2$

This yields limits on the deviations of the data from the Rosenbluth fit of roughly 0.4% (0.7%) for the elastic (inelastic), assuming $\Delta\varepsilon$ range of 0.7

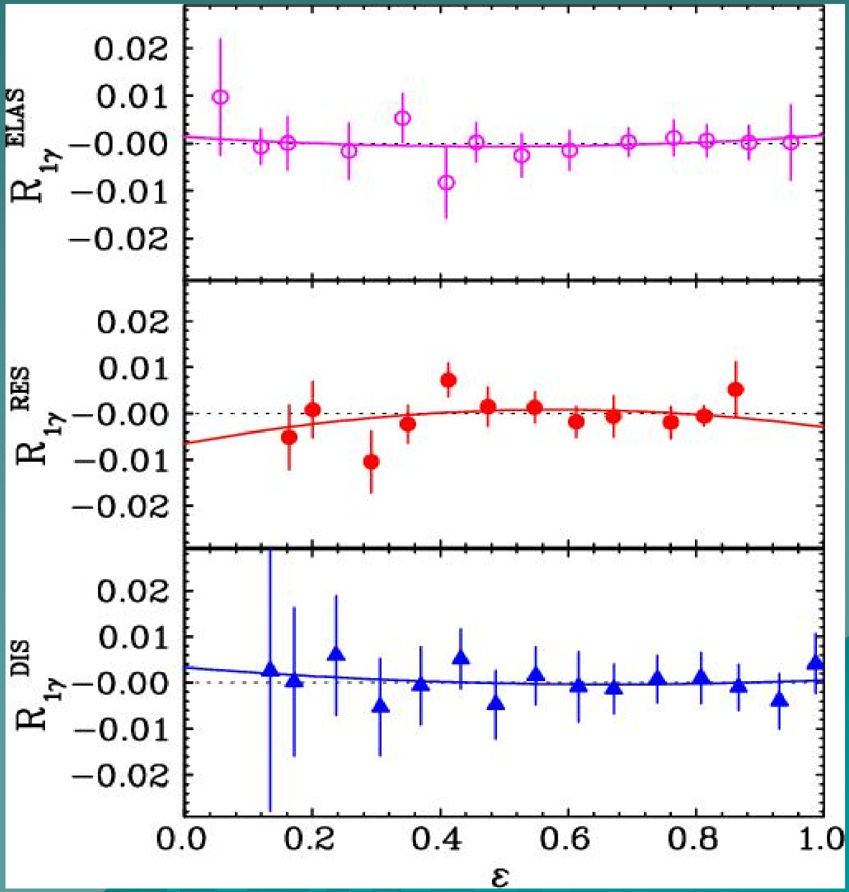
$$R_{1\gamma} = \frac{\sigma_{Data} - \sigma_{fit}}{\sigma_{fit}}$$

In the absence of TPE contributions, one expects $R_{1\gamma} = 0$ in every ε bin

$$R_{1\gamma} = A + B\left(\varepsilon - \varepsilon_0\right)^2$$

$A \leq 0.05\ \%$

Region	B
Elastic	(0.9+/-2.0)%
Res. region	(-2.3+3/0)%
Inelastic Region	(0.9+/-3.8)%



While the limits in table provide the best quantitative limits on deviations from linearity, the residuals on the right plot give a better idea of the sensitivity of the different data sets in different regions of ε.

Summary

Experiment e99-118

Analysis of the experiment e99-118 is finished for Hydrogen and Deuterium Targets. R Does Not go to zero (as Q^2 goes to zero), only at very low x there is a hint that R goes to zero. Analysis indicates that possibly $R^H > R^D$. Still some work to do with radiative corrections for heavy targets.

Hydrogen & Deuterium results will be published shortly.

Experiment e00-02

Analysis of the experiment e00-02 is on the way. Calibrations, efficiency calculations are completed. Still some work to do with CSB and Rad. Corrections.

Experiment e94-110

Analysis of the experiment e94-110 is finished. Structure Functions F_1, F_L, R are determined with a high precision in the Resonance region.

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Two Photon-Effects

We do not find any evidence for TPE effects. The 95% confidence level upper limit on the curvature parameter P2, was found to be 6.4% (10.6%) for the elastic (inelastic) data. This limits maximum deviations from a linear fit to ≤ 0.4% (0.7%).

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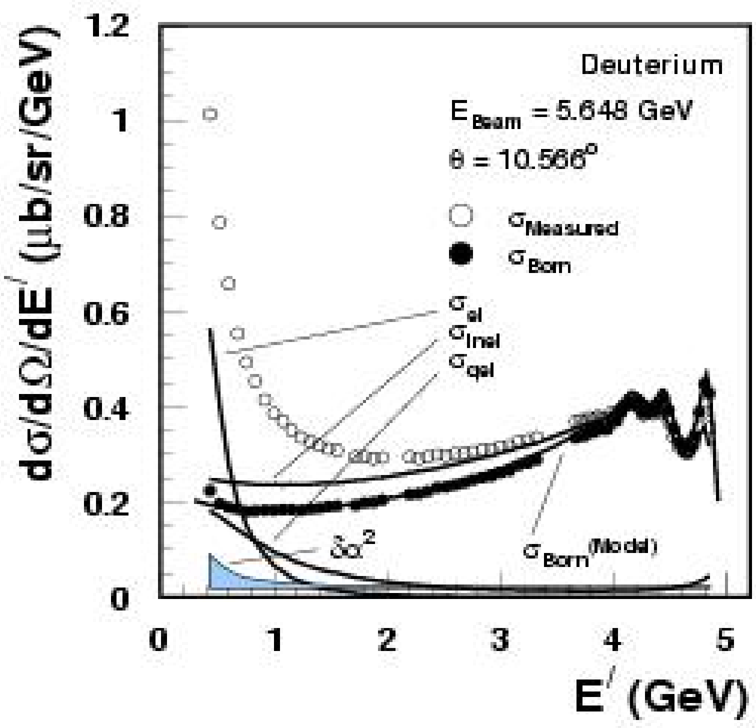
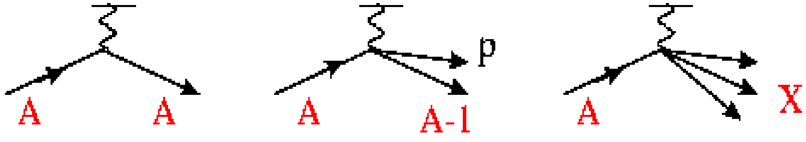
Radiative Corrections





$$\sigma_{Born} = \left(\sigma_{Meas} - \sigma_{El} - \sigma_{Qel}\right)\frac{\sigma_{Born}^{Model}}{\sigma_{Inel}}$$

Bardin: (TERAD) Only calculates Internal Radiative Corrections (Includes 2-photon Corrections)

Mo,Tsai: calculates Internal & External Radiative Corrections

$$\sigma_{Int} = \sigma_{El}^{Int} + \sigma_{Qel}^{Int} + \sigma_{Inel}^{Int}$$

$$\sigma_{Ext} = \sigma_{El}^{Ext} + \sigma_{Qel}^{Ext} + \sigma_{Inel}^{Ext}$$

$$\sigma_{Born} = \sigma_{Meas} - \sigma_{Bardin}^{Int}\left(\frac{\sigma^{Int} + \sigma^{Ext}}{\sigma^{Int}}\right)_{Mo,Tsai}$$

$$R_{e99118}^{A>2}\left(Q^2\right)=\left(\frac{A\times Q^2}{Q^4+B}\right)+\langle C\rangle$$